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Steady-State Response of a Well-Balanced Wire Pair to Distributed Interference

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We show that the longitudinal circuit defined by a well-balanced wire pair in a cable can be studied independently of the metallic circuit. The metallic circuit is then excited by the longitudinal voltage and current acting through the wire pair and terminal unbalances. Discrete parameter longitudinal circuits are defined which have the same terminal response as the distributed-parameter longitudinal circuit. These equivalent circuits are studied under an electrically short assumption, yielding simple expressions for their terminal response. The electrically short assumption enables the distributed impressed voltage which excites the longitudinal circuit to be represented by only two parameters, the "total impressed voltage" and the "center of impressed voltage." These parameters are analogous to the total mass and center of mass of a thin filament or wire. Finally, an analysis of a longitudinal circuit defined by a subscriber loop excited by a nearby power distribution system is used to derive a relationship between the short-circuit longitudinal current at the central office and the open-circuit longitudinal voltage at the telephone set. This relationship is used to estimate the distribution of short-circuit longitudinal current at the central office from a known distribution of open-circuit longitudinal voltage at the telephone set.

I. INTRODUCTION

The problem of computing the steady-state response of a multiconductor system has received considerable attention in the literature. Carson and Hoyt[1] developed the classical transmission line equations and S. O. Rice[2] developed the mathematical techniques necessary for their solution. Even so, the complexities introduced by a large number of conductors tend to limit the amount of basic understanding of fundamental problems, such as the effects of longitudinal induction and longitudinal-to-metallic conversion, that can be obtained by pursuing the multiconductor problem. Moreover, a large number of conductors are necessarily described by a large number of parameters upon which there is often a paucity of data.

We present an in-depth analysis of the steady-state response of a well-balanced wire pair to distributed interference. This simplification of the general problem enables the analysis to continue beyond the formal solution to develop both an intuitive feel for the problem and a simple model for use in engineering applications. The effect of other pairs can be approximated in the one-pair model by a judicious choice of model parameters.

Historically, the primary emphasis has been on characterizing the longitudinal and metallic voltage at the subscriber's telephone set because of the impact of these voltages on the quality of the communication's path.[6,7] More recently, the use of electronic loop terminating equipment has generated interest in the longitudinal current at the central office. This study was motivated by the need to characterize the longitudinal current at the central office and to better understand the roles that the terminal and wire pair unbalances play in longitudinal-to-metallic conversion.

II. SUMMARY OF RESULTS

We begin with the classical transmission line equations which define the steady-state response of the wire pair when excited by a distributed impressed voltage. A transformation to the longitudinal and metallic voltages and currents is employed to study the longitudinal and metallic circuits defined by the wire pair. We show that if the wire pair and its terminations are "well-balanced," then the longitudinal circuit (LC) can be studied independently of the metallic circuit (MC). The MC is then excited by the longitudinal voltage and current acting through the wire pair and terminal unbalances. The unbalances admit the following interpretations: (*i*) longitudinal current flowing through the distributed impedance unbalance of the wire pair can be represented as a distributed series voltage generator in the MC, (*ii*) longitudinal voltage across the distributed admittance unbalance of the wire pair can be represented as a distributed shunt current generator in the MC, and (*iii*) longitudinal current through a discrete impedance unbalance in a termination can be represented as a discrete voltage generator in the termination for the MC.

The response of the LC is studied in some detail. Discrete parameter circuits are defined which have the same terminal response as the distributed parameter LC. These equivalent circuits are studied under an electrically short assumption, yielding simple expressions for their terminal response. The electrically short assumption enables the distributed

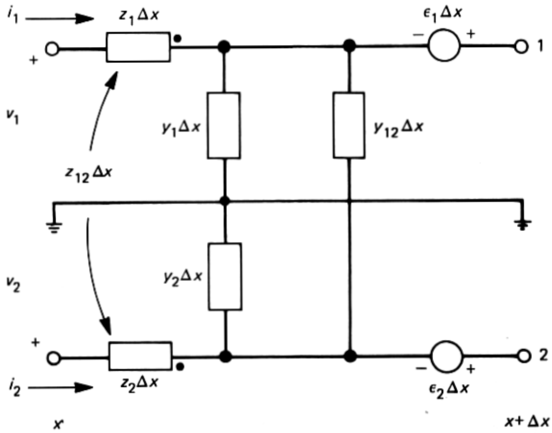


Fig. 1—Incremental circuit model.

impressed voltage to be represented by only two parameters, the total impressed voltage and the center of impressed voltage. These parameters are analogous to the total mass and center of mass of thin filament or wire. Finally, an analysis of the LC defined by a subscriber loop excited by a nearby power distribution system is used to derive a relationship between the short-circuit longitudinal current at the central office and the open-circuit longitudinal voltage at the telephone set. This relationship is used to estimate the distribution of short-circuit longitudinal current at the central office from the known distribution of open-circuit longitudinal voltage at the telephone set.

III. BASIC EQUATIONS

3.1 Transmission line equations

The transmission line equations for a wire pair in a cable excited by a distributed impressed voltage acting as a fixed radian frequency are

$$v_1'(x) = -z_1(x)i_1(x) - z_{12}i_2(x) + \epsilon_1(x)$$

$$i_1'(x) = -[y_1(x) + y_{12}]v_1(x) + y_{12}v_2(x) \tag{1}$$

$$v_2'(x) = -z_2(x)i_2(x) - z_{12}i_1(x) + \epsilon_2(x)$$

$$i_2'(x) = -[y_2(x) + y_{12}]v_2(x) + y_{12}v_1(x). \tag{2}$$

Boundary conditions, discussed in Section 3.2, are determined from terminations at the ends ($x = 0$ and ℓ) of the wire pair. The above equations are essentially Rice's[2] eqs. (1.1) and (1.2) except for our notation which is motivated by the incremental circuit model of Fig. 1. Notice that three types of unbalances are possible at x; an impedance unbalance $[z_1(x) \neq z_2(x)]$, an admittance unbalance $[y_1(x) \neq y_2(x)]$ and an unbalance in the impressed voltage $[\epsilon_1(x) \neq \epsilon_2(x)]$.

The metallic and longitudinal voltages and currents are defined as follows:

$$v(x) = v_1(x) - v_2(x) = \text{metallic voltage}$$

$$i(x) = \frac{1}{2}[i_1(x) - i_2(x)] = \text{metallic current} \tag{3}$$

$$v_g(x) = \frac{1}{2}[v_1(x) + v_2(x)] = \text{longitudinal voltage}$$

$$i_g(x) = i_1(x) + i_2(x) = \text{longitudinal current.} \tag{4}$$

Transforming the transmission line equations [eqs. (1) and (2)] to the metallic and longitudinal voltages and currents yields

$$v'(x) = -zi(x) + \delta_z(x)i_g(x) + \delta_\epsilon(x)$$

$$i'(x) = -yv(x) + \delta_y(x)v_g(x) \tag{5}$$

$$v_g'(x) = -z_g i_g(x) + \delta_z(x)i(x) + \epsilon_g(x)$$

$$i_g'(x) = -y_g v_g(x) + \delta_y(x)v(x). \tag{6}$$

The parameters of eqs. (5) and (6) are defined in terms of the parameters of eqs. (1) and (2) and Fig. 1 as follows: The impedances in ohms per unit length are

$$z = z_1(x) + z_2(x) - 2z_{12}$$

$$z_g = \frac{1}{4}[z_1(x) + z_2(x) + 2z_{12}]$$

$$\delta_z = \frac{1}{2}[z_2(x) - z_1(x)]. \tag{7}$$

The admittances in mhos per unit length are

$$y = \frac{1}{4}[y_1(x) + y_2(x)] + y_{12}$$

$$y_g = y_1(x) + y_2(x)$$

$$\delta_y(x) = \frac{1}{2}[y_2(x) - y_1(x)]. \tag{8}$$

The impressed voltages in volts per unit length are

$$\epsilon_g(x) = \frac{1}{2}[\epsilon_1(x) + \epsilon_2(x)]$$

$$\delta_\epsilon(x) = \epsilon_1(x) - \epsilon_2(x). \tag{9}$$

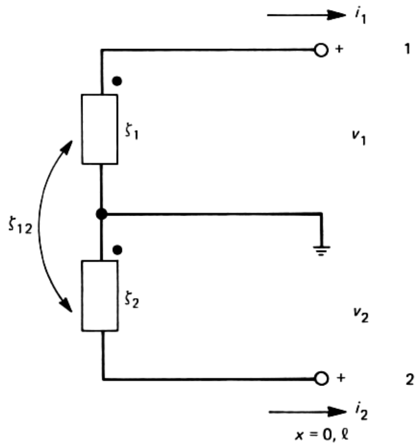


Fig. 2—Termination circuit model.

Only the unbalances, which now appear explicitly, and the impressed voltages remain x-dependent; all other parameters are assumed uniform.

3.2 Boundary conditions

The boundary conditions necessary to determine a particular solution to eqs. (1) and (2), or equivalently eqs. (5) and (6), are determined from the terminations at the ends of the wire pair. Consider the canonical passive-symmetric termination of Fig. 2. The boundary conditions for the voltages and currents relative to ground are

$$\begin{aligned} v_1 &= -\zeta_1 i_1 - \zeta_{12} i_2 \\ v_2 &= -\zeta_2 i_2 - \zeta_{12} i_1. \end{aligned} \tag{10}$$

Transforming to the metallic and longitudinal voltages and currents [eqs. (3) and (4)] yields

$$v = -\zeta i + \Delta_\zeta i_g \tag{11}$$

$$v_g = -\zeta_g i_g + \Delta_\zeta i. \tag{12}$$

The parameters of eqs. (11) and (12) are defined in terms of the parameters of eq. (10) and Fig. 2 as follows:

$$\begin{aligned} \zeta &= \zeta_1 + \zeta_2 + 2\zeta_{12} \\ \zeta_g &= \frac{1}{4}(\zeta_1 + \zeta_2 - 2\zeta_{12}) \\ \Delta_\zeta &= \frac{1}{2}(\zeta_2 - \zeta_1). \end{aligned} \tag{13}$$

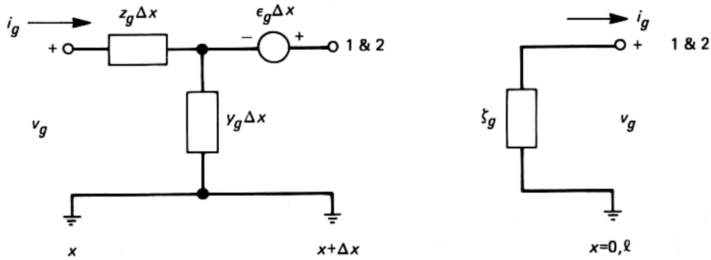


Fig. 3—Longitudinal circuit model.

3.3 Longitudinal and metallic circuits

The metallic and longitudinal voltages and currents supported by a wire pair are determined by eqs. (5) and (6) together with boundary conditions of the type given in eqs. (11) and (12) at the ends of the wire pair. Notice that if the wire pair and its terminations are perfectly balanced, then the metallic voltage and current are identically zero. Hence it follows by continuity[3] that if the wire pair and its terminations are well-balanced (i.e., the unbalances are small relative to their associated longitudinal parameters*), then the metallic voltage and current are small relative to the longitudinal voltage and current.

Under a well-balanced assumption, the second-order terms in eqs. (6) and (12) can be neglected, leaving

$$\begin{aligned} v_g'(x) &= -z_g i_g(x) + \epsilon_g(x) \\ i_g'(x) &= -y_g v_g(x) \end{aligned} \tag{14}$$

with boundary conditions of the form

$$v_g = -\zeta_g i_g. \tag{15}$$

Notice that the longitudinal voltage and current can be assumed independent of both the metallic voltage and current and the system unbalances. Moreover, the circuit models of Fig. 3 which represent the above equations define the LC.

Now assume that $v_g(x)$ and $i_g(x)$ are known in eqs. (5) and (11). Then the metallic voltage and current satisfy

$$\begin{aligned} v'(x) &= -zi(x) + \epsilon(x) \\ i'(x) &= -yv(x) + \xi(x) \end{aligned} \tag{16}$$

with boundary conditions of the form

* Cable pairs have an average resistance unbalance of 2 percent and an average capacitance unbalance to ground of 0.5 percent.

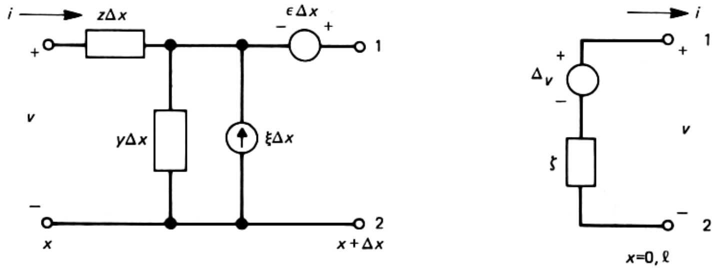


Fig. 4—Metallic circuit model.

$$v = -\zeta i + \Delta_\zeta i_g \tag{17}$$

where

$$\epsilon(x) = \delta_z(x) i_g(x) + \delta_\epsilon(x)$$
$$\xi(x) = \delta_y(x) v_g(x). \tag{18}$$

are all known. The circuit models in Fig. 4 represent the above equations and define the MC. Note that the x-dependence of the wire pair unbalances pose no analytical problems since they appear in the forcing functions $\epsilon(x)$ and $\xi(x)$. Moreover, superposition can be applied to yield a decomposition of the metallic voltage and current as a sum of terms due to each unbalance acting separately.

Let us summarize our results. A well-balanced wire pair with well-balanced terminations defines a longitudinal and a metallic circuit. The LC can be assumed independent of the MC. The MC is excited by the longitudinal voltage and current acting through the wire pair and terminal unbalances which can be interpreted as follows: longitudinal current $i_g(x)$ following through the distributed impedance unbalance of the wire pair $\delta_z(x)$ can be represented as a distributed series voltage generator $\delta_z(x) i_g(x)$ in the MC, longitudinal voltage $v_g(x)$ across the distributed admittance unbalance of the wire pair $\delta_y(x)$ can be represented as a distributed shunt current generator $\delta_y(x) v_g(x)$ in the MC, and longitudinal current i_g flowing through the discrete impedance unbalance δ_ζ in a termination can be represented as a discrete voltage generator $\Delta_\zeta i_g$ in the termination for the MC. Finally, superposition can be applied to yield a decomposition of the metallic voltage and current as a sum of terms due to each unbalance acting separately. The equations describing the LC and MC, since they have constant coefficients, are amenable to standard techniques[3] which are applied to analyze the LC in the next section.

IV. LONGITUDINAL CIRCUIT ANALYSIS

4.1 Discrete-parameter equivalent circuits

Equation (14) describing the response of the LC defined by a well-balanced wire pair is conveniently represented as a forced linear system,

$$\begin{bmatrix} v_g(x) \\ i_g(x) \end{bmatrix}' = -\begin{bmatrix} 0 & z_g \\ y_g & 0 \end{bmatrix} \begin{bmatrix} v_g(x) \\ i_g(x) \end{bmatrix} + \begin{bmatrix} \epsilon_g(x) \\ 0 \end{bmatrix}. \tag{19}$$

The boundary conditions defined by the longitudinal impedances ζ_g^0 and ζ_g^ℓ of the well-balanced terminations at the ends ($x = 0$ and ℓ) of the wire pair are, from eq. (15),

$$\begin{aligned} v_g(0) &= -\zeta_g^0 i_g(0) \\ v_g(\ell) &= \zeta_g^\ell i_g(\ell). \end{aligned} \tag{20}$$

The solution of eq. (19) is of the form

$$\begin{bmatrix} v_g(\ell) \\ i_g(\ell) \end{bmatrix} = \Phi_g(\ell) \begin{bmatrix} v_g(0) \\ i_g(0) \end{bmatrix} + \int_0^\ell \Phi_g(\ell - \xi) \begin{bmatrix} \epsilon_g(\xi) \\ 0 \end{bmatrix} d\xi. \tag{21}$$

Since the LC is assumed uniform (i.e., z_g and y_g are independent of x), the transition matrix of the LC, $\Phi_g(\xi)$, can be expressed in terms of the characteristic impedance $k_g = \sqrt{z_g/y_g}$ and propagation constant $\gamma_g = \sqrt{z_g y_g}$ of the LC,

$$\Phi_g(\xi) = \begin{bmatrix} \cosh \gamma_g \xi & -k_g \sinh \gamma_g \xi \\ -\dfrac{1}{k_g} \sinh \gamma_g \xi & \cosh \gamma_g \xi \end{bmatrix}. \tag{22}$$

Substituting the boundary conditions [eq. (20)] and the above expression for $\Phi_g(\xi)$ in eq. (21) yields after some manipulation

$$\begin{bmatrix} \zeta_g^0 \cosh \gamma_g \ell + k_g \sinh \gamma_g \ell & \zeta_g^\ell \\ \zeta_g^0 \sinh \gamma_g \ell + k_g \cosh \gamma_g \ell & -k_g \end{bmatrix} \begin{bmatrix} i_g(0) \\ i_g(\ell) \end{bmatrix} = \begin{bmatrix} \displaystyle\int_0^\ell \epsilon_g(\xi) \cosh \gamma_g(\ell - \xi) d\xi \\ \displaystyle\int_0^\ell \epsilon_g(\xi) \sinh \gamma_g(\ell - \xi) d\xi \end{bmatrix}. \tag{23}$$

The formal solution of eq. (23) for $i_g(0)$ and $i_g(\ell)$ is of the form

$$i_g = \frac{E_g}{\zeta_g + Z_g} \tag{24}$$

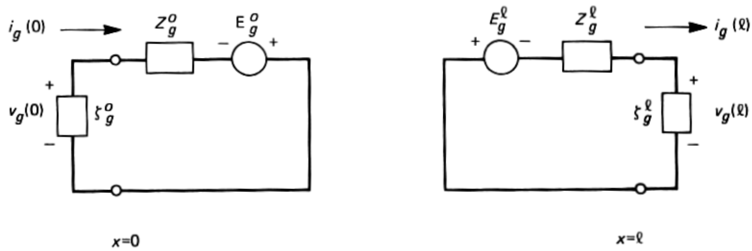


Fig. 5—Discrete-parameter longitudinal circuit models.

where E_g is the longitudinal source and Z_g is the longitudinal impedance seen looking into the LC from one end, terminated in ζ_g at the other end. These quantities define two discrete parameter circuits (see Fig. 5) which have the same terminal response as the distributed parameter circuit of Fig. 3.

General expressions for E_g and Z_g at each end of the LC are given below:

$$E_g^0 = \frac{\displaystyle\int_0^\ell \epsilon_g(\xi)\left[\cosh \gamma_g(\ell-\xi) + \frac{\zeta_g^\ell}{k_g}\sinh \gamma_g(\ell-\xi)\right] d\xi}{\cosh \gamma_g\ell + \dfrac{\zeta_g^\ell}{k_g}\sinh \gamma_g \ell} \tag{25}$$

$$E_g^\ell = \frac{\displaystyle\int_0^\ell \epsilon_g(\xi)\left[\cosh \gamma_g\xi + \frac{\zeta_g^0}{k_g}\sinh \gamma_g\xi\right] d\xi}{\cosh \gamma_g\ell + \dfrac{\zeta_g^0}{k_g}\sinh \gamma_g \ell} \tag{26}$$

$$Z_g^0 = \frac{\zeta_g^\ell + k_g \tanh \gamma_g\ell}{1 + \dfrac{\zeta_g^\ell}{k_g}\tanh \gamma_g\ell} \tag{27}$$

$$Z_g^\ell = \frac{\zeta_g^0 + k_g \tanh \gamma_g\ell}{1 + \dfrac{\zeta_g^0}{k_g}\tanh \gamma_g\ell}. \tag{28}$$

These expressions simplify if the LC is terminated in its characteristic impedance, k_g. For example, setting $\zeta_g^\ell = k_g$ in eqs. (25) and (27) yields

$$E_g^0 = \int_0^\ell \epsilon_g(\xi)e^{-\gamma_g\xi}d\xi$$

$$Z_g^0 = k_g. \tag{29}$$

Note that the voltage impressed furthest from $x = 0$ suffers the most attenuation as one would intuitively expect.

4.2 The electrically short longitudinal circuit

An LC is electrically short if its electrical length, $|\gamma_g \ell|$, is small. This is typically the case when the source of impressed voltage is a nearby power distribution system. If $|\gamma_g \ell|$ is small, then the hyperbolic functions can be approximated by the first terms of their power series expansions; $\sinh \gamma_g \ell \approx \gamma_g \ell$, $\cosh \gamma_g \ell \approx 1$, and $\tanh \gamma_g \ell \approx \gamma_g \ell$. These approximations when substituted into eqs. (25)–(28) yield approximations of the longitudinal source and longitudinal impedance,

$$\hat{E}_g^0 = E \frac{1 + \zeta_g^\ell y_g (\ell - \overline{\ell})}{1 + \zeta_g^\ell y_g \ell} \tag{30}$$

$$\hat{E}_g^\ell = E \frac{1 + \zeta_g^0 y_g \overline{\ell}}{1 + \zeta_g^0 y_g \ell} \tag{31}$$

$$\hat{Z}_g^0 = \frac{\zeta_g^\ell + z_g \ell}{1 + \zeta_g^\ell y_g \ell} \tag{32}$$

$$\hat{Z}_g^\ell = \frac{\zeta_g^0 + z_g \ell}{1 + \zeta_g^0 y_g \ell}. \tag{33}$$

The quantities E and $\overline{\ell}$ are defined as the total impressed voltage,

$$E = \int_0^\ell \epsilon_g(\xi) d\xi \tag{34}$$

and the center of impressed voltage,

$$\overline{\ell} = \frac{1}{E} \int_0^\ell \xi \epsilon_g(\xi) d\xi. \tag{35}$$

These definitions have a physical intrepretation if the impressed voltage is a real valued and nonnegative function. In this case, $E \geq 0$ and $0 \leq \overline{\ell} \leq \ell$ and the center of impressed voltage can be interpreted as that point along the LC where the impressed voltage may be concentrated without changing the terminal response of the LC. Hence a particular E and $\overline{\ell}$ define a class of equivalent impressed voltages whose canonical representative is a point source of strength E based at $\overline{\ell}$. These concepts have their mechanical analogs in the total mass and the center of mass of a thin filament or wire. Recall that the center of mass is that point along the wire where the total mass may be concentrated while preserving moments about the ends of the wire. In our case, the center of impressed voltage is that point along the LC where the impressed voltage may be concentrated while preserving the response at the ends of the LC.

V. ENGINEERING APPLICATION

5.1 The longitudinal response of an electrically short subscriber loop

The most common LC is a subscriber loop excited by a nearby power distribution system. This circuit is typically electrically short at the fundamental frequency of the impressed voltage (60 Hz). Longitudinally, a subscriber loop has a low impedance termination at the central office by virtue of the battery supply circuit, and essentially an open-circuit termination at the telephone set, assuming single-party or isolated ringer service. Consequently, the longitudinal quantities of primary interest are the short-circuit current at the central office, I_g, and the open-circuit voltage at the telephone set, V_g.

We now derive a simple relationship between these two fundamental quantities. To begin, I_g can be expressed as

$$I_g = \frac{\hat{E}_g^0}{\hat{Z}_g^0} \tag{36}$$

where $\hat{E}_g^0$ and $\hat{Z}_g^0$ are given for an arbitrary ζ_g^ℓ in eqs. (30) and (32). Letting $\zeta_g^\ell \to \infty$ in these equations yields

$$\hat{E}_g^0 = \frac{E(\ell - \overline{\ell})}{\ell} \tag{37}$$

$$\hat{Z}_g^0 = \frac{1}{y_g \ell}. \tag{38}$$

Substitution of these results into eq. (36) yields

$$I_g = E y_g(\ell - \overline{\ell}). \tag{39}$$

Similarly, V_g can be expressed as

$$V_g = \hat{E}_g^\ell \tag{40}$$

where $\hat{E}_g^\ell$ is given in eq. (31) for an arbitrary ζ_g^0. Setting $\zeta_g^0 = 0$ yields

$$V_g = E. \tag{41}$$

Hence I_g and V_g satisfy

$$I_g = V_g y_g(\ell - \overline{\ell}). \tag{42}$$

The link between V_g and I_g is the center of exposure, $\overline{\ell}$.

5.2 Estimating V_g

Historically, emphasis has been placed on characterizing the open-circuit longitudinal voltage at the telephone set because it can be converted to an interfering metallic voltage (V) across the telephone set if unbalances are present in the loop.[7] The historical measure of loop balance[6] is

$$\text{BAL} = 20 \log \left| \frac{V_g}{V} \right| \qquad \text{dB.} \tag{43}$$

The balance of the loop determines the amount of longitudinal voltage that will be converted to metallic voltage by the loop unbalances. This can be expressed mathematically as

$$N = N_g - \text{BAL.} \tag{44}$$

The metallic circuit noise (N) and the longitudinal circuit noise (N_g, usually called noise to ground) are defined as

$$N = 20 \log \left| \frac{V}{V_R} \right| \qquad \text{dBrn} \tag{45}$$

$$N_g = 20 \log \left| \frac{V_g}{V_R} \right| \qquad \text{dBrn.} \tag{46}$$

The reference noise (dBrn) voltage is 24.5 μV which corresponds to 1 pW across a 600 Ω resistor. The commonly used 3-type noise measuring set attenuates a longitudinal noise measurement by 40 dB. Hence 40 dB must be added to a measured value to obtain N_g in dBrn.

The distributions of noise and loop balance for Bell System loops were determined in 1964 as part of the General Loop Survey of physical and transmission characteristics of the loop plant.[4] Noise measurements were made on 1100 randomly selected loops during normal working hours at the subscriber's telephone set using both 3-kHz flat and C-message frequency weighting. C-message weighting approximates the frequency response of the telephone set and the human ear, and 3-kHz flat-weighting assigns equal weight to all frequencies in the 0 to 3 kHz band. Since the primary interferer on subscriber loops is a nearby power distribution system with a fundamental frequency of 60 Hz, a 3-kHz flat measurement is dominated by the 60-Hz component. Hence the rms voltage corresponding to N_g measured with 3-kHz flat weighting is essentially 60 Hz. The distribution of this voltage over the 1100 loops is shown in Fig. 6. The maximum voltage was 18 V. Ninety-nine percent of the loops had a measured voltage of less than 11 V. The average voltage was 1.5 V with a standard deviation of 2.1 V.

5.3 Estimating I_g

The range of longitudinal current at the central office affects both the operation of existing loop terminating equipment and the design of new equipment. In this section, we estimate the distribution of longitudinal

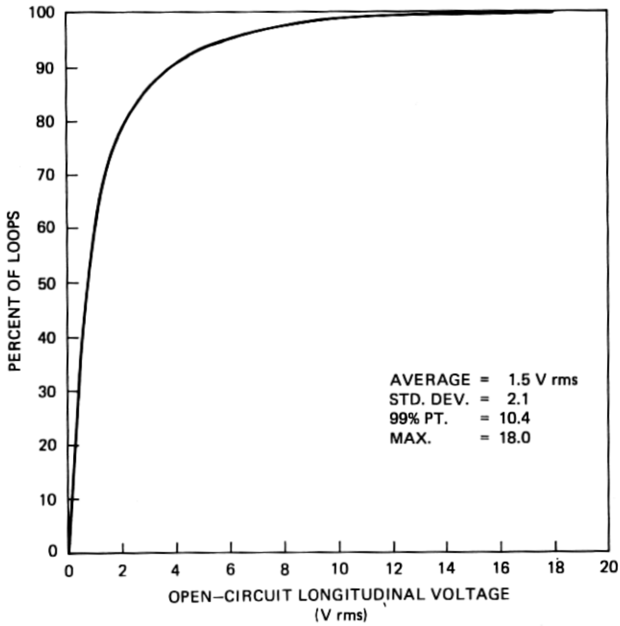


Fig. 6—Distribution of open-circuit rms longitudinal voltage at the telephone set (1964 General Loop Survey).

current using the distribution of longitudinal voltages and loop lengths determined as part of the 1964 General Loop Survey.

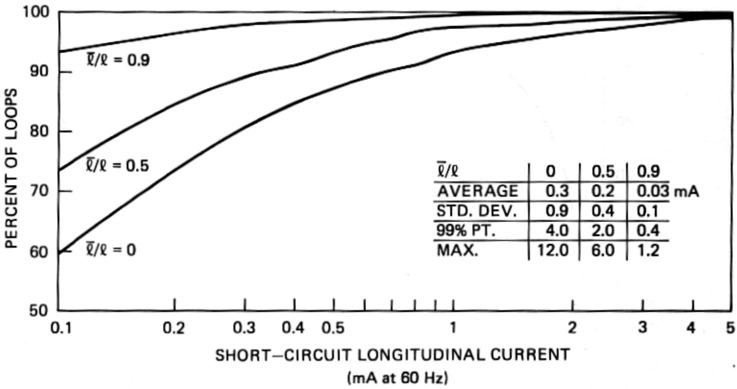
The basic equation [eq. (42)] relating I_g and V_g can be expressed in the form

$$|I_g| = |V_g||y_g|\ell(1 - \overline{\ell}/\ell). \tag{47}$$

The ratio $\overline{\ell}/\ell$ is a measure of where the exposure is centered along the loop. If the exposure is centered at the subscriber ($\overline{\ell}/\ell = 1$), then the short-circuit current at the central office is zero. Conversely, the current is maximum if the exposure is centered over the central office ($\overline{\ell}/\ell = 0$). In this case,

$$\max|I_g| = |V_g||y_g|\ell.$$

Equation (47) can be used to estimate the distribution of $|I_g|$ from the distributions of $|V_g|$ (see Fig. 6) and ℓ (see Fig. 2 of Ref. 4) and an assumed value of $\overline{\ell}/\ell$. The admittance of the longitudinal circuit is primarily capacitive at 0.17 μF per mi. Distributions of $|I_g|$ at 60 Hz for $\overline{\ell}/\ell$ = 0, 0.5, and 0.9 are shown in Fig. 7. In the worst-case condition of $\overline{\ell}/\ell$ = 0, the maximum current is 12 mA. Ninety-nine percent of all loops had a current of less than 4 mA. The average current was 0.3 mA with a



$\overline{\ell}/\ell$	0	0.5	0.9
AVERAGE	0.3	0.2	0.03 mA
STD. DEV.	0.9	0.4	0.1
99% PT.	4.0	2.0	0.4
MAX.	12.0	6.0	1.2

Fig. 7—Estimated distributions of short-circuit 60-Hz longitudinal current at the central office (1964 General Loop Survey).

standard deviation of 0.9 mA. If the exposures are centered at the midpoints of the loops ($\overline{\ell}/\ell = 0.5$), then the above currents are reduced by a factor of 2. If the exposures are centered near the subscriber ($\overline{\ell}/\ell = 0.9$), then the above currents are reduced by a factor of 10, i.e., the maximum current reduces to 1.2 mA and the average current reduces to 0.03 mA.

5.4 Estimating $\overline{\ell}/\ell$

The estimated distribution of the short-circuit longitudinal current at the central office is very sensitive to where the exposures are centered along the loops. An estimate of $\overline{\ell}/\ell$ can be made from simultaneous measurements of $|I_g|$ and $|V_g|$ using another form of eq. (47),

$$\overline{\ell}/\ell = 1 - \frac{|I_g|}{|V_g|\,|y_g|\,\ell}. \tag{48}$$

A limited number of near-simultaneous measurements were made as part of the CO Strata Survey described in Ref. 5. Single near-simultaneous measurements of $|I_g|$ and $|V_g|$ at 60 Hz were made during normal working hours for each of the 47 test loops. These data were not discussed in Ref. 5 but were made available to us by D. N. Heirman. The ratio $\overline{\ell}/\ell$ was calculated for each test loop using eq. (48). The distribution of the calculated values over the 47 loops is shown in Fig. 8. All exposures were centered beyond the midpoints of the loops. The average value of $\overline{\ell}/\ell$ was 0.88. Hence the exposure of the average loop was centered at almost 90 percent of the loop length. These results are intuitively pleasing since the worst exposures (long aerial parallels with single-phase) are indeed

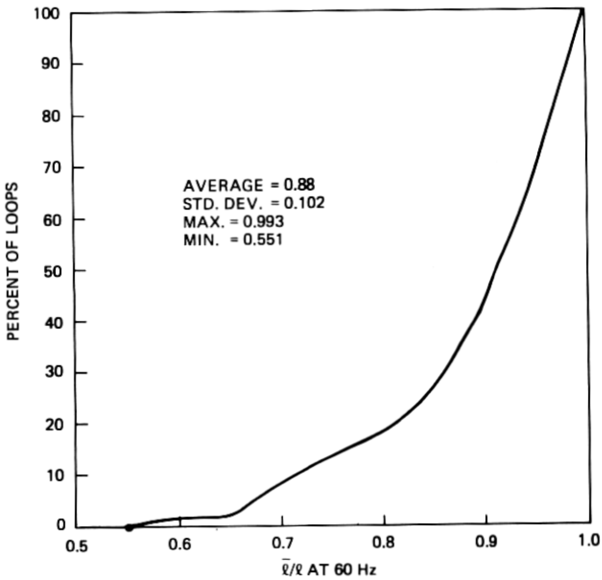


Fig. 8—Distribution of $\overline{\ell}/\ell$ at 60 Hz (Central Office Strata Survey).

over the far ends of the loops. If this distribution of $\overline{\ell}/\ell$ is representative, then the worst case condition is not $\overline{\ell}/\ell = 0$ but $\overline{\ell}/\ell = 0.5$. In this case (see Fig. 7), the maximum current at 60 Hz. is 6 mA, 99 percent of all loops had a current of less than 2 mA and the average current was 0.15 mA.

5.5 Use of the data

The data presented in this section are based on one-time measurements made during normal working hours as part of the 1964 General Loop Survey. The demand for commercial power has doubled since 1964, and one would expect the induced voltages to have increased as a result. In addition, recent surveys[5] have shown that time-of-day variation in 60-Hz induction is likely to be 2 to 1. In residential areas in particular, the induction is likely to be higher in the evening than during the day when the 1964 measurements were made. For these reasons, the reader is cautioned to use the data presented in this section with care.

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